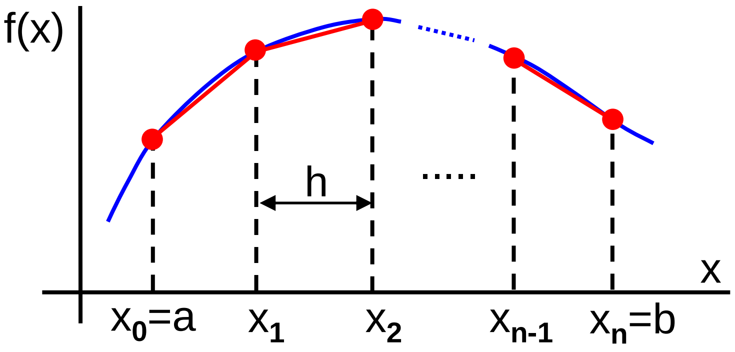
Composite 1/3 Simpson Rule



In general we have n+1 points and n intervals.

If the points are equispaced h = (b-a)/n

If there are even number of points the integration can not be done with Simpson's 1/3 rule only.

$$I = \int_a^b f(x)\,dx = \int_{x0}^{x2} f(x)\,dx + \int_{x2}^{x4} f(x)\,dx + \ldots + \int_{x(n-2)}^{xn} f(x)\,dx$$

$$I \approx \frac{h}{3}\left[f(x_0)+4f(x_1)+f(x_2)\right] + \frac{h}{3}\left[f(x_2)+4f(x_3)+f(x_4)\right] + \ldots + \frac{h}{3}\left[f(x_{n-2})+4f(x_{n-1})+f(x_n)\right]$$

$$I \approx (b-a)\left[\frac{f(x_0) + 4\sum_{i=1,3,5}^{n-1} f(x_i) + 2\sum_{i=2,4,6}^{n-1} f(x_i) + f(x_n)}{3n}\right]$$

$$E_a = -\frac{(b-a)^5}{180\,n^4}\,\bar{f}^{(4)}(x)$$

Example Calculate $\int_0^{\pi} \sin(x)\,dx$ using

(a) Trapezoidal Rule with n=2, n=4 and n=6.

(b) Simpson's 1/3 Rule with n=2, n=4 and n=6.

(a) For n=2, h=π/2,

$$I \approx \frac{h}{2}\left[f(x_0) + 2f(x_1) + f(x_2)\right] = \frac{\pi/2}{2}\left[\sin(0) + 2\sin(\pi/2) + \sin(\pi)\right] = \frac{\pi}{2} = 1.570796$$

$$E_a = -\frac{(b-a)^3}{12n^2}\bar{f}''(x) = -\frac{(\pi-0)^3}{12(2)^2}\frac{\int_0^{\pi} -\sin(x)\,dx}{\pi-0} = 0.411234$$

Note that $E_t = 0.429204$

(b) For n=4, h=π/4,

$$I \approx \frac{h}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\right]$$

$$= \frac{\pi/4}{3}\left[\sin(0) + 4\sin(\pi/4) + 2\sin(\pi/2) + 4\sin(3\pi/4) + \sin(\pi)\right] = 2.004560$$

$$E_a = -\frac{(b-a)^5}{180n^4}\bar{f}^{(4)}(x) = -\frac{(\pi-0)^5}{180(4)^4}\frac{\int_0^{\pi} \sin(x)\,dx}{\pi-0} = 0.004228$$

Note that $E_t = 0.004560$