Cubic Interpolation

Often, values in a table change in such a way that linear interpolation is not suitable. Cubic interpolation uses the values of four adjacent table entries (e.g., at x_0, x_1, x_2 and x_3) to obtain the coefficients of the cubic equation $y = a + bx + cx^2 + dx^3$ to use as an interpolating function between x_1 and x_2. For example, to find the freezing point for a 33.3 wt% solution of ethylene glycol using cubic interpolation requires the four table values in Figure 5-12 whose x values are highlighted.

A convenient way to perform cubic interpolation is by means of the Lagrange fourth-order polynomial

$$y_x = \frac{(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)} y_1 + \frac{(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)} y_2$$

$$+ \frac{(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)} y_3 + \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)} y_4 \qquad (5\text{-}2)$$

	A	B	C	D
1	Freezing and Boiling Points of Heat Transfer Fluid			
2	Wt% Ethylene Glycol	Freezing Point, °F	Boiling Point, °F (at 1 atm)	Refractive Index (at 22°C)
16	29.0	8.0	219	1.3624
17	30.0	6.7	220	1.3635
18	31.0	5.4	220	1.3646
19	32.0	4.2	220	1.3656
20	33.0	2.9	220	1.3667
21	34.0	1.4	220	1.3678
22	35.0	-0.2	221	1.3688
23	36.0	-1.5	221	1.3699
24	37.0	-3.0	221	1.3709
25	38.0	-4.5	221	1.3720
26	39.0	-6.4	221	1.3730

Figure 5-12. Four bracketing x values required to perform cubic interpolation at $x = 33.3\%$.
(folder 'Chapter 05 Interpolation', workbook 'Interpolation I', sheet Cubic Interpolation')

The Lagrange fourth-order polynomial is cumbersome to use in a worksheet function, but convenient to use in the form of a custom function. A compact and elegant implementation of cubic interpolation in the form of an Excel 4.0 Macro Language custom function was provided by Orvis*. A slightly modified version, in VBA, is provided here (Figure 5-13). The syntax of the custom function is InterpC(***lookup_value, known_x's, known_y's***). The argument *lookup_value* is the value of the independent variable for which you want the interpolated *y* value; *known_x's* and *known_y's* are the arrays of independent and dependent variables, respectively, that comprise the table. The table must be sorted in ascending order of *known_x's*.

---

* William J. Orvis, *Excel 4 for Scientists and Engineers*, Sybex Inc., Alameda, CA, 1993.

```
Function InterpC(lookup_value, known_x's, known_y's)
' Performs cubic interpolation, using an array of known_x's, known_y's.
' The known_x's must be in ascending order.
' Based on XLM code from Excel for Chemists", page 239,
' which was based on W. J. Orvis' code.

Dim row As Integer
Dim i As Integer, j As Integer
Dim Q As Double, Y As Double

row = Application.Match(lookup_value, known_x's, 1)
   If row < 2 Then row = 2
   If row > known_x's.Count - 2 Then row = known_x's.Count - 2

For i = row - 1 To row + 2
   Q = 1
For j = row - 1 To row + 2
   If i <> j Then Q = Q * (lookup_value - known_x's(j)) / (known_x's(i) - _
known_x's(j))
Next j
   Y = Y + Q * known_y's(i)
Next i
InterpC = Y
End Function
```

Figure 5-13. Cubic interpolation function procedure.
(folder 'Chapter 05 Interpolation', workbook 'Interpolation I', module 'CubicInterpolation')

Figure 5-14 illustrates the use of the custom function to interpolate values in the table shown in Figure 5-12; cell H22 contains the formula

=InterpC(G22,$A$3:$A$47,$B$3:$B$47)

	F	G	H	I
20	Using a custom function for cubic interpolation			
21		wt%	FP, °F	
22		33.3	2.47	

Figure 5-14. Using the InterpC function procedure for cubic interpolation.
(folder 'Chapter 05 Interpolation', workbook 'Interpolation I', sheet 'Linear Interpolation')

Cubic Interpolation in a Table by Using the TREND Worksheet Function

In the TREND function, the array *known_x's* can include one or more sets of independent variables. For example, suppose column A contains x values. You can enter x^2 values in column B and x^3 in column C and then regress columns A through C against the y values in column D to obtain a cubic interpolation function. But instead of actually entering values of the square and the cube of the x values, you can use an array constant in an array formula, thus

```
{=TREND(C19:C22,A19:A22^{1,2,3},F9^{1,2,3},1)}
```

This example of using the TREND function is found in folder 'Chapter 05 Interpolation', workbook 'Interpolation I', sheet Cubic Interpolation').