Filters and smoothers

- One-dimensional datasets or time series datasets often are a superposition of many wave harmonics
- In some cases, we want to smooth the data or remove high frequency (small wave) components
- This is known as low-pass filtering

The Shapiro filter (also called a digital filter)

- A simple technique is known as the Shapiro filters.
- This involves adding a value at grid point i, and also adding values of surrounding points, then taking a weighted average. The value at i has the most weight.
- This reduces the contribution of smaller wavelengths – especially $2\Delta x$ waves and $4\Delta x$ waves.
- Repeating this exercises essentially removes the contributions of smaller wavelengths, but also gradually removes the influence of longer wavelengths. It results in a smoothed solution.

Some Shapiro filter equations

$$\Gamma_i = \frac{1}{4}(\psi_{i-1} + 2\psi_i + \psi_{i+1})$$

$$\Gamma_i = \frac{1}{16}(-\psi_{i-2} + 4\psi_{i-1} + 10\psi_i + 4\psi_{i+1} - \psi_{i+2})$$

$$\Gamma_i = \frac{1}{64}(\psi_{i-3} - 6\psi_{i-2} + 15\psi_{i-1} + 44\psi_i + 15\psi_{i+1} - 6\psi_{i+2} + \psi_{i+3})$$

$$\Gamma_i = \frac{1}{256}(-\psi_{i-4} + 8\psi_{i-3} - 28\psi_{i-2} + 56\psi_{i-1} + 186\psi_i$$
$$+ 56\psi_{i+1} - 28\psi_{i+2} + 8\psi_{i+3} - \psi_{i-4})$$

These can be generalized in stencil form as

TABLE 2. *Stencils for $f_i^{(p+1)}$ for Various Values of p*

p		f_i	$f_{i\pm 1}$	$f_{i\pm 2}$	$f_{i\pm 3}$	$f_{i\pm 4}$	$f_{i\pm 5}$	$f_{i\pm 6}$	$f_{i\pm 7}$	$f_{i\pm 8}$	$f_{i\pm 9}$	$f_{i\pm 10}$
0	$1/2^2$	(2	1)									
1	$1/2^4$	(10	4	− 1)								
2	$1/2^6$	(44	15	− 6	1)							
3	$1/2^8$	(186	56	− 28	8	− 1)						
4	$1/2^{10}$	(772	210	− 120	45	− 10	1)					
5	$1/2^{12}$	(3172	792	− 495	220	− 66	12	− 1)				
6	$1/2^{14}$	(12952	3003	− 2002	1001	− 364	91	− 14	1)			
7	$1/2^{16}$	(52666	11440	− 8008	4368	− 1820	560	− 120	16	− 1)		
8	$1/2^{18}$	(213524	43758	− 31824	18564	− 8568	3060	− 816	153	− 18	1)	
9	$1/2^{20}$	(863820	167960	− 125970	77520	− 38760	15504	− 4845	1140	−190	20	− 1)

where, for example, p=1 is $1/2^4$ is 4^{th} order (n=2)

The response function

- By including more neighboring points, the removal of small Δx waves is enhanced while retaining the amplitudes of the longer Δx for each pass.

- However, its more difficult to deal with the endpoints unless it's a periodic domain.

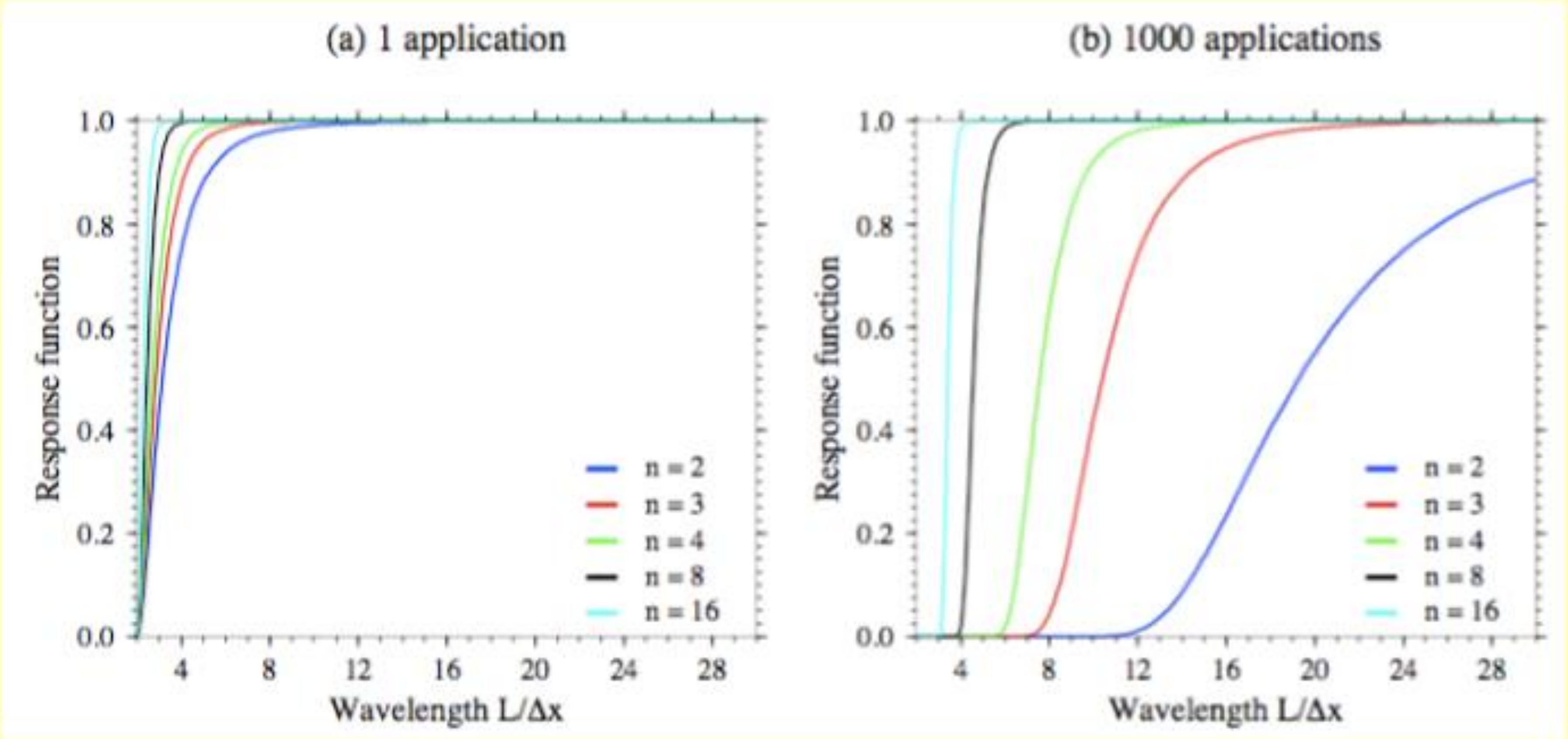
- The damping effect for each pass can be quantified by the response function R. R varies from 0 to 1, where 1 is no damping and 0 is complete damping of the original value.

- For the Shapiro filter, the response function is:

$$R_n(k) = 1 - sin^{2n}\left(k\frac{\Delta x}{2}\right) = 1 - sin^{2n}(\pi\frac{\Delta x}{L})$$

where 2n is the "order"

Digital filters: Response function



Note that the simplest Shapiro filter (the 1-2-1 filter) has a strong local damping effect. It should be used carefully.

- Response function of different Shapiro filters after (a) 1 application and (b) 1000 applications. $2n$ indicates the order of the Shapiro filter. Higher orders need more data points.